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Abstract
With increasing demand of long term applications, supercapacitors have been widely chosen as energy storage devices for energy harvesting aware wireless sensor networks (WSNs) due to their long charging-discharging life cycles. However, few studies have focused on charge redistribution effect of supercapacitors in WSNs. In this paper, we investigate charge redistribution of supercapacitor and explore how it affects long term energy neutral operation (ENO). The Variable Leakage Resistance (VLR) model is used to analyze the charge redistribution effect. Our results indicate that charge redistribution may cause considerable amount of extra energy loss in long term ENO. A practical algorithm to minimize charge redistribution loss during energy neutral operation is proposed. The algorithm is computationally lightweight and can be incorporated into the state-of-the-art duty cycling power management strategies in WSNs. The proposed algorithm is proved to be effective in keeping the main branch and the delayed branch balanced and thus lowering energy dissipation from charge redistribution.

Categories and Subject Descriptors
C.3 [Special-Purpose and Application-Based Systems]: Real-time and embedded systems

General Terms
 Algorithms, Systems, Performance

Keywords
 Supercapacitors, Charge redistribution, WSN, Energy-Neutral Systems

1 Introduction
Wireless sensor networks are pervasive in modern society, including smart building systems [18], environmental monitoring systems [11] and wearable human body sensor networks [13]. To extend WSN lifetime, great efforts have been made to develop energy harvesting aware WSNs and related power management strategies. However, traditional rechargeable batteries restrain the lifetime of a WSN due to their limited cycle life. Typically, rechargeable batteries have limited lifetimes of 500 to 1200 cycles. By the end of the cycle life, rechargeable batteries will suffer about 20% capacity loss and the useful energy drops to 50% because of higher internal resistance. Supercapacitors, whose cycle life is usually more than 500,000 cycles, are considered to be promising energy buffer devices for harvesting aware systems.

Existing research works focus on building wireless sensor systems with supercapacitor or supercapacitor-rechargeable battery hybrid [4] energy storage systems. Those works mainly solve the problems of system design, including Maximum Power Point Tracking (MPPT) [10], voltage/current regulator design [9] and alternative ambient power sources (solar [3], wind [8] and etc).

Others explore dynamic power management algorithms to better utilize harvested energy [2]. Energy neutral operation (ENO) algorithms are investigated to show possibility of maintenance-free perpetual operation [5]. To satisfy the ENO condition, adaptive duty cycling algorithms [7, 14] are adopted to match load energy consumption to the ambient energy harvesting rate.

However, in previous supercapacitor-related literatures, researchers either treat supercapacitors as ideal storage devices or use empirical leakage models [19] to characterize the devices. There have been few literatures focusing on charge redistribution in supercapacitor-operated WSNs. According to recent studies [16], supercapacitors feature self discharge and charge redistribution. Both phenomenons may lead to significant energy loss, especially in long term WSN applications. Compared with self discharge, charge redistribution of supercapacitor has not been well studied in wireless sensor networks.

In this paper, we use the VLR model, which has been previously validated by experiments [15, 17], to show that accumulated energy dissipated by supercapacitor charge redistribution can be considerably high in long term ENO. A new power management algorithm is designed to reduce charge redistribution loss. The algorithm can be easily integrated with the state-of-the-art power management methods to better maintain the ENO condition. The proposed method is evaluated by simulation, and the results demonstrate its ef-
2 Charge Redistribution of Supercapacitor
2.1 Equivalent Circuit Model of Supercapacitors

A supercapacitor is made up of two porous electrodes immersed in electrolyte and separated by one porous insulating membrane. Its physical structure increases the Farad value as well as the complexity of accurate modeling. The VLR model we use is a simplified equivalent circuit model for power management research [16].

As shown in Figure 1, the VLR model features two resistor-capacitor branches. Each branch includes an equivalent capacitor and a resistor. In the first branch, the equivalent capacitor $C_1$ includes a constant capacitor $C_0$ and a voltage dependent capacitor $K_1 \cdot V_{C1}$. The other branch consists of a constant capacitor $C_2$. The variable resistor $R_3$ is related to self discharge. The rated voltage $V_{nom}$ denotes the highest voltage the supercapacitor can be charged to.

Table 1 lists the VLR model parameter values of supercapacitors of different size: 10 $F$ and 50 $F$. The variable $R_3$ is modeled as a piece-wise linear function changes with terminal voltage $V_{sc}$, as shown in Eq(1) and (2) for the 10 $F$ and 50 $F$ supercapacitors, respectively.

$$R_3 = \begin{cases} 
(-2.969 \cdot V_{sc} + 8.043) \cdot 10^6 & 2.68 \leq V_{sc} \leq 2.7 \\
(-5.515 \cdot V_{sc} + 14.87) \cdot 10^6 & 2.662 \leq V_{sc} < 2.68 \\
(-5.821 \cdot V_{sc} + 15.66) \cdot 10^6 & 0 \leq V_{sc} < 2.662 
\end{cases}$$

$$R_3 = \begin{cases} 
(-1.942 \cdot V_{sc} + 5.291) \cdot 10^6 & 2.666 \leq V_{sc} \leq 2.7 \\
(-2.340 \cdot V_{sc} + 6.354) \cdot 10^5 & 2.625 \leq V_{sc} < 2.666 \\
(-3.656 \cdot V_{sc} + 9.566) \cdot 10^5 & 0 \leq V_{sc} < 2.625 
\end{cases}$$

From Eq(5), we can see that the larger difference between $V_{C1}$ and $V_{C2}$, the higher charge redistribution power is. Figure 2 illustrates the quadratic relationship between $|V_{C1} - V_{C2}|$ and charge redistribution power $P_{chd}$. Eq(5) indicates that smaller $R_2$ leads to larger charge redistribution power for a fixed voltage difference between $V_{C1}$ and $V_{C2}$. The supercapacitor with higher rated capacitance tends to have smaller $R_2$ as shown in Table 2, which causes higher charge redistribution loss with the same voltage difference.

3 Impact of Charge Redistribution on Power Management in WSN

In this section, we further discuss how charge redistribution affects long term ENO. Simulation is used to quantitatively analyze charge redistribution loss.

3.1 VLR-based Supercapacitor Updating Model

In order to keep track of long term supercapacitor charge redistribution, we establish an algorithm to update supercapacitor state of charge (SoC). Assume $I_{chd}$ is the charging current and $I_{con}$ is the load current. Since supercapacitor SoC is related to its charging and discharging history, algorithm 1 is proposed to update $V_{C1}$ and $V_{C2}$ iteratively and calculate $P_{chd}$.

In the first branch, $C_1$ consists of a constant capacitor and a voltage-dependent capacitor, which means $C_1$ changes with
the terminal voltage $V_c$. However, algorithm 1 approximates $C_1$ using a constant in one simulation time step. The approximation requires the time step to be small enough. Here, we use 5 ms as the time step.

### 3.2 Simulation Setup for Harvesting Aware Wireless Sensor node

In order to evaluate the impact of charge redistribution in wireless sensor networks, we simulate a wireless sensor node to quantitatively analyze the accumulated dissipated energy caused by charge redistribution.

**ENO Condition:** The simulated system is tuned to energy neutral operation, which means the average harvesting rate $\overline{P_{\text{har}}}$ should be approximated to the average energy consuming rate $\overline{P_{\text{con}}}$.

$$\overline{P_{\text{har}}} \approx \overline{P_{\text{con}}} \quad (6)$$

**Energy Harvester:** To simplify simulation, the charging power profile (Harvesting power profile with MPPT) is assumed to be periodic and pulse-wise. The duration $t_d$ and amplitude $P_{\text{pulse}}$ of the pulse are determined by the average harvesting rate. $K$ represents the number of pulses in one period $T$, which is set to be 1 in our case. We use phase to indicate when the charging pulse will show up in one period $T$. It is not easy to obtain the phase of harvesting pulse ahead of time.

$$P_{\text{pulse}} \cdot t_d \cdot K = \overline{P_{\text{har}}} \cdot T \quad (T \gg t_d) \quad (7)$$

**Energy Storage Device:** The sensor node uses a single supercapacitor as energy buffer. The terminal voltage of supercapacitor ranges from 0 to $V_{nom}$. The initial SoC of the simulated supercapacitor is set to be $V_{sc} = V_{C1} = V_{C2}$.

**Algorithm 1: VLR based supercapacitor simulation model**

1. **Initialization**: $V_{sc}(t) \leftarrow V_{ac}(t_0)$
2. $V_{C1}(t) \leftarrow V_{C1}(t_0)$
3. $V_{C2}(t) \leftarrow V_{C2}(t_0)$
4. **Loop begin**
5. $t \leftarrow t_k$
6. $I_{R1}(t_k) = I_{ch}(t_k) - I_{con}(t_k) - \frac{V_{ac}(t_k) - V_{ac}(t_{k-1})}{K}$
7. $I_{R2}(t_k) = \frac{V_{ac}(t_k) - V_{ac}(t_{k-1})}{R_2}$
8. $V_{C1}(t_k) = V_{C1}(t_{k-1}) + (t_k - t_{k-1}) \cdot I_{R1}$
9. $V_{C2}(t_k) = V_{C2}(t_{k-1}) + (t_k - t_{k-1}) \cdot I_{R2}$
10. $V_{ac}(t_k) = V_{C1}(t_k) + I_{R1}(t_k) \cdot R_1$
11. $P_{chd}(t_k) = \frac{V_{C1}(t_k) - V_{C2}(t_k)}{R_2}$
12. $k \leftarrow k + 1$
13. **End loop**

### Table 1. VLR Model Parameters of Different Supercapacitors

<table>
<thead>
<tr>
<th>Capacitance (F)</th>
<th>Manufacturer</th>
<th>$V_{nom}$ (V)</th>
<th>$R_1$ ($\Omega$)</th>
<th>$C_0$ (F)</th>
<th>$K_v$ ($V/\sqrt{F}$)</th>
<th>$R_2$ ($\Omega$)</th>
<th>$C_2$ (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Maxwell</td>
<td>2.7</td>
<td>0.067</td>
<td>7.278</td>
<td>2.136</td>
<td>139.34</td>
<td>1.9137</td>
</tr>
<tr>
<td>50</td>
<td>Maxwell</td>
<td>2.7</td>
<td>0.014</td>
<td>35.193</td>
<td>13.773</td>
<td>33.76</td>
<td>11.850</td>
</tr>
</tbody>
</table>

### Figure 3. (a) Charging power profile (b) Discharging power profile

**Interface Circuit:** The system uses a boost DC-DC converter to regulate supercapacitor voltage. The startup voltage of the converter is 0.7 V and the output is 3.3 V. The conversion efficiency $\eta$ is modeled to be 80%, which means:

$$P_{\text{con}} = \begin{cases} 0 & 0 \leq V_{sc} < 0.7 \\ \frac{P_{\text{node}}}{\eta} & 0.7 \leq V_{sc} \leq V_{nom} \end{cases} \quad (8)$$

**Wireless Sensor Node:** The simulated sensor node is assigned with periodic tasks. The average power consumption of sensor node is controlled by its duty cycle $D$. The node has only two working modes, which are active mode with power $P_{\text{active}}$ and sleep mode with $P_{\text{sleep}}$. The nominal voltage of sensor node is 3.3 V. $t_{sch}$ is defined as the time when the sensor node starts to work in active mode in one period $T$. $t_{ex}$ denotes the active time in one period, $t_{ex} = T \cdot D$. The average consuming rate in the sensor node is:

$$\overline{P_{\text{node}}} = P_{\text{active}} \cdot D + (1 - D) \cdot P_{\text{sleep}} \quad (0 \leq D \leq 1) \quad (9)$$

The instantaneous sensor node power in one period ($t \in [0,T]$) is:

$$P_{\text{node}} = \begin{cases} 0 & 0 \leq V_{sc} < 0.7 \\ P_{\text{active}} & (0.7 \leq V_{sc} \leq V_{nom}) \text{ and } (t_{sch} \leq t \leq \text{t}_{sch} + \text{t}_{ex}) \\ P_{\text{sleep}} & (0.7 \leq V_{sc} \leq V_{nom}) \text{ and } (t < \text{t}_{sch} \text{ or } t > \text{t}_{sch} + \text{t}_{ex}) \end{cases} \quad (10)$$

Figure 3(a) shows the charging profile in one period ($T = 100$ s). The phase of the charging pulse is 0 s and the $t_d$ is 10 s. The amplitude of charging pulse is 1 W. Figure 3(b) shows that $t_{sch}$ of the sensor node is 90 s. The amplitude of discharging pulse is also 1 W. The sensor node achieves energy neutral operation. The active function time $t_{ex}$ is 10 s that gives 10% duty cycle.

### 3.3 Simulation Results

Two simulation scenarios are investigated.
Simulation 1: The phase of charging pulse is deterministic. The sensor node sticks to lazy scheduling to avoid possible energy deficit. \( T = 100 \, s \). Table 2 shows the simulation results without any power management algorithm. To satisfy the energy neutral operation, we set \( P_{\text{har}} = 1 \, W, t_d = 10 \, s, \) \( P_{\text{active}} = 0.8 \, W, P_{\text{sleep}} = 0.1 \, mW, D = 10\% \). One thing needs to be pointed out that although the node is assigned with 10\% duty cycle, due to charge redistribution loss and other energy dissipation, the actual duty cycle will be less than 10\%. We use the actual active time to calculate the average duty cycle metric.

From Table 2, we can see that in long term operation, charge redistribution dissipated energy will be accumulated. It causes significant energy loss and deficit. For one week, the total dissipated charge redistribution energy is about 160 \( J \) for the 10 \( F \) supercapacitor. Considering the simulated sensor node, 160 \( J \) can support the node to work for 33 minutes with 10\% duty cycle. For a 10 \( F \) supercapacitor, the maximum amount of usable energy is about 34 \( J \). The charge redistribution loss in one week is about three times more than its storage capacity.

Simulation 2: Simulation 1 gives an ideal case with fixed charging pulse phase. However, phase is never fixed in real situation. For most of the off-the-shelf harvesters, it is hard to have full clairvoyance of harvested energy. We can only get an estimation of average power harvesting rate over a long time. Furthermore, when harvesters are placed in changeable environment, it is even impossible for us to get the average harvesting rate. A lot of practical issues cause unpredictable effect, such as weather, ambient objects activity(human, animal) and shadowing. Simulation 2 explores charge redistribution with random charging pulse phase. Table 3 gives detailed simulation results.

The average energy harvesting rate in simulation 1 and 2 are exactly the same. So are the node power consumptions. However, by comparing the energy loss data in Table 2 and Table 3, we find that the dissipated energy caused by charge redistribution in simulation 2 is more than three times of the value in simulation 1. To further investigate the detailed charging and discharging process, Figure 4 and Figure 5 show a snapshot from \( t = 500 \, s \) to \( 1000 \, s \). From Figure 4, due to the deterministic charging pulse phase and lazy scheduling policy, the voltage difference between \( V_1 \) and \( V_2 \) appears to be periodic. For most of the time, the voltage difference is approximately 0.1 \( V \). However, in simulation 2, the charging pulse phase is a pure random number, which leads to a changeable \( |V_{\text{c1}} - V_{\text{c2}}| \). We can see that for a large portion of time, the difference is about 0.3 to 0.4 \( V \), as shown in Figure 5. In Figure 2, we already demonstrated that larger \( |V_{\text{c1}} - V_{\text{c2}}| \) voltage difference will cause larger charge redistribution power.

4 Power Management for Reducing Charge Redistribution Loss

Section 3 has shown that due to charge redistribution, the lazy scheduling is not effective in a supercapacitor-powered sensor node with random charging pulse. It raises an interesting task scheduling problem. For a supercapacitor-buffered wireless sensor node, how to better schedule tasks to reduce charge redistribution loss? In this section, we propose a new algorithm which fully makes use of wireless sensor node hardware and software to reduce energy loss due to charge redistribution. The proposed algorithm is compared with the state-of-the-art duty cycle adapting algorithm using simulation.

4.1 Energy Efficient Algorithm to Reduce Charge Redistribution

Under the energy neutral condition, an intuitive way to reduce charge redistribution is to start task execution right after the charging pulse, which can keep the value of \( |V_{\text{c1}} - V_{\text{c2}}| \) small. Then the problem is converted to be how to track charging pulse phase. Following the intuition, we propose a new task scheduling algorithm to solve the problem.

Algorithm 2 details how tasks are scheduled in one period. The main idea is to combine software and hardware to track harvested energy. It makes use of the external interrupt pin (INT) onchip to detect rising edge of charging pulses. Then the scheduler sets an analog-to-digital converter(ADC) to scan supercapacitor terminal voltage every \( t_s \) s. If the harvested energy is sufficient, tasks are scheduled. If there is no charging pulse or insufficient harvested energy, tasks are suspended until there is no slack time, like lazy scheduling. When there is no slack time, the deadline handler will respond and schedule the related tasks.

The algorithm assumes that the system has clairvoyance of task set in terms of execution time and deadline in the beginning of each period. For long term WSN applications,
most of tasks are cyclic data collection and transfer. It is reasonable to assume knowledge of workload.

The proposed algorithm attempts to minimize energy overhead while reducing charge redistribution. A typical MCU used in WSNs, such as MSP430 [12], consumes about 10 mA in active mode and less than 1 mA in sleep mode. So, the proposed algorithm puts MCU into sleep whenever it is possible. The overhead contained in the algorithm are periodic ADC sampling, timer operation and simple calculation. All these operations are simple and instantaneous, which guarantees that the precious MCU resources are not over-occupied by task scheduler.

In terms of the hardware requirement, INT, timer and ADC sampling pins are available in MCUs of most commercial wireless sensor systems. An 8-bit microcontroller [1] is enough to execute algorithm 2. In terms of software, most popular embedded Operating Systems are capable of implementing interrupt service, timer control and ADC reading. Taking TinyOS [6] as an example, which is the most widely used OS in WSNs, interrupt, timer and ADC reading have already been interfaced into system APIs. Therefore, algorithm 2 can be implemented on any wireless sensor node system that is equipped with a common MCU and capable of running TinyOS.

Clearly, when the charging profile is pulse-wise, which meets our assumptions, algorithm 2 is capable of tracking pulses by using edge triggered INT. On the other hand, when the charging profile turns out to be smooth, even like constant charging power, algorithm 2 still remains effective. In line 12 of algorithm 2, ADC is periodically called to track supercapacitor terminal voltage change. In this way, harvested energy can be reflected by the change of \( V_{sc} \) through Eq(11). Once the harvested energy is sufficient, tasks are scheduled to reduce \( |V_{C1} - V_{C2}| \). Therefore, without loss of generality, the proposed algorithm 2 achieves charge redistribution reduction by tracking harvested energy.

\[
E_{bar} = \Delta E_{sc} + \bar{P}_{con} \cdot (t_k - t_{k-1}) \\
= \frac{1}{2} \{ V_{sc}^2(t_k) - V_{sc}^2(t_{k-1}) \} + \bar{P}_{con} \cdot (t_k - t_{k-1})
\]  

(11)

### 4.2 Evaluation of the Proposed Algorithm

Simulation is used to demonstrate the effectiveness of the proposed algorithm. The assumptions of the ENO condition and the periodic pulse-wise harvesting profile are still applied in the simulation. For simplicity and consistency, we use the setup made in Section 3. The storage device used in simulation is a 10 F Maxwell supercapacitor.

For comparison, a lifetime-based feed-back controlled adaptive duty cycling algorithm [7, 19] is also implemented. The duty cycling algorithm has no clairvoyance of charge redistribution. The results are also compared with that in Table 3, which is treated as no power management case.

From Table 4 we can see, compared with no power management, algorithm 2 can effectively cut down charge redistribution loss in all cases. The magnitude of energy loss reduction grows with increasing simulation time. In one hour, algorithm 2 reduces half of the loss. In one day, the value is about 88 % and in one week, the reduction reaches up to 94%. Compared with the adaptive duty cycling algorithm, algorithm 2 achieves more than 30% of energy loss reduction at all time scales. Without clairvoyance of charge redistribution, the adaptive duty cycling algorithm takes conservative actions when adjusting duty cycles, which explains why it has the lowest average duty cycle in all scenarios. The proposed charge redistribution reduction algorithm can be incorporated into the state-of-the-art power management algorithm to further improve overall performance. The hybrid algorithm can further lower energy loss. In one week,
Table 4. Power management to reduce charge redistribution

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Energy loss(J)</th>
<th>Avg Duty Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Energy loss(J)</td>
<td>Avg Duty Cycle</td>
</tr>
<tr>
<td>No power management</td>
<td>3.458</td>
<td>6.099</td>
</tr>
<tr>
<td>Adaptive duty cycling</td>
<td>1.726</td>
<td>38.142</td>
</tr>
<tr>
<td>Algorithm 2</td>
<td>1.241</td>
<td>7.899</td>
</tr>
<tr>
<td>Hybrid(Alg. 2 and Duty cycling)</td>
<td>1.18</td>
<td>5.486</td>
</tr>
</tbody>
</table>

its charge redistribution energy is only 50% and 79% of the loss of original adaptive duty cycling and algorithm 2 respectively. In addition, with capability of adaptive duty cycling, the hybrid algorithm outperforms the original algorithm 2 with lower redistribution loss. By integrating algorithm 2 into adaptive duty cycling, we make use of the methods in Section 3 to estimate the charge redistribution power, which enables the lifetime predictor to be clairvoyant of charge redistribution. The hybrid algorithm has the lowest charge redistribution energy loss and the highest average duty cycle.

In WSN applications, duty cycling algorithms are essential to adjust wireless sensor system to respect the energy neutral operation condition. Typically, duty cycling algorithms have high computational cost. A combination of the lightweight charge redistribution reduction algorithm and state-of-the-art duty cycling algorithm is proved to work well in keeping energy neutral operation.

5 Conclusions

In this paper, we investigate charge redistribution in supercapacitor-operated wireless sensor nodes. The simulation results show that the dissipated energy caused by charge redistribution can be significant in long-term energy neutral systems. To the best of our knowledge, this is the first work to quantitatively analyze long-term energy dissipation due to charge redistribution of supercapacitor.

To reduce charge redistribution loss, we propose a low-complexity task scheduling algorithm. The algorithm is suitable for general energy harvesting situations. It does not require prediction of harvested energy. The proposed algorithm fully utilizes hardware and software resources in a wireless sensor node to track harvested power and then execute tasks timely. In addition, the proposed algorithm can collaborate with the state-of-the-art duty cycling algorithm for power management in WSNs with energy neutral operations. In long-term energy neutral operation, the hybrid algorithm can reduce charge redistribution energy loss to only 5% of that without power management and 50% of that with original duty cycling algorithm without incurring significant MCU operation overhead.

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7 References